Spring 2014 **Math 601** Name: ____________________

Quiz 7

Question 1. (4 pts)

Determine whether each of the following statements is true or false. You do NOT need to explain.

(a) If A is an $n \times n$ matrix with all entries being integers, then $\det(A)$ is also an integer.

Solution: True.

(b) If A is an $n \times n$ real matrix, then all its eigenvalues are real numbers.

Solution: False.

Question 2. (6 pts)

Use elementary row operations to compute the determinant of

$$A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 1 & 6 & 4 & 8 \\ 1 & 3 & 0 & 0 \\ 2 & 6 & 4 & 12 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 1 & 6 & 4 & 8 \\ 1 & 3 & 0 & 0 \\ 2 & 6 & 4 & 12 \end{bmatrix} \xrightarrow{\text{add a multiple of } R_1 \text{ to other rows}} \begin{bmatrix} 1 & 3 & 2 & 4 \\ 0 & 3 & 2 & 4 \\ 0 & 0 & -2 & -4 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Notice that the operation "adding a multiple of a row to another row" does not change the determinant. So $\det(A) = 1 \cdot 3 \cdot (-2) \cdot 4 = -24$.

Question 3. (10 pts)

(a) Find all eigenvalues and eigenvectors of $B = \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix}$.

(b) Determine whether B is diagonalizable.

Solution:

(a) The characteristic polynomial is

$$\det(tI_2 - B) = \begin{bmatrix} t-3 & 5 \\ -1 & t+1 \end{bmatrix} = (t-3)(t+1) + 5 = t^2 - 2t + 2$$

Solve $t^2 - 2t + 2 = 0$ and we get eigenvalues $t = 1 \pm i$.

When $t = 1 + i$, solve

$$\begin{bmatrix} -2+i & 5 \\ -1 & 2+i \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

We get an eigenvector of $v_1 = \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$.

Similarly, for $t = 1 - i$, we get an eigenvector $v_2 = \begin{pmatrix} 2-i \\ 1 \end{pmatrix}$.

(b) B is a 2×2 matrix with 2 linearly independent eigenvectors, so B is diagonalizable.